Automated Ambiguity Detection in Layout-Sensitive Grammars

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[*] This work was done at Tsinghua University.

What Are Layout-Sensitive Grammars?

Grammars where indentations and whitespaces affect parsing.

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if False:
    print(1, end=' ')
print(2)
# Output: 2
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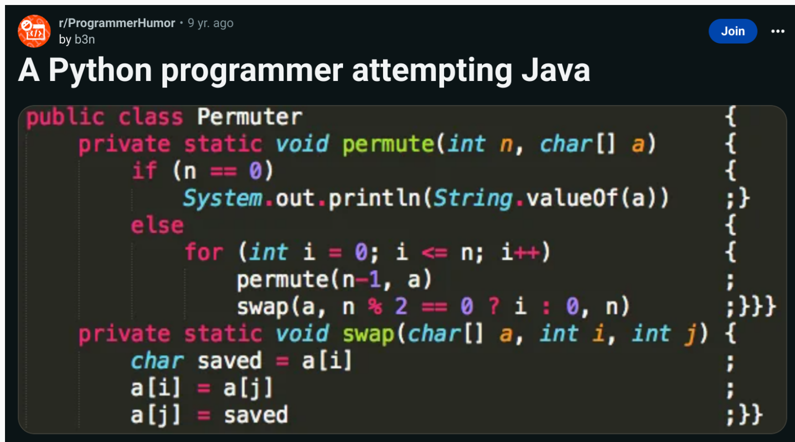

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That programmer would love Scala 3!

```
object Permuter:
  private def permute(n: Int, a: Array[Char]): Unit =
    if n == 0 then
      println(String.valueOf(a))
    else
      for i <- 0 to n do
        permute(n-1, a)
        swap(a, if n % 2 == 0 then i else 0, n)
  private def swap(a: Array[Char], i: Int, j: Int) =
    val saved = a(i)
    a(i) = a(j)
    a(j) = saved
```

Ambiguity Matters

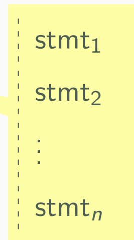
A useful new layout-sensitive grammar should be unambiguous.

Manually Checking Ambiguity is Hard

Let's consider a layout-sensitive grammar:

$$\text{block} \rightarrow \|\text{stmt}\|^{+}$$

$$\text{stmt} \rightarrow \textbf{nop} \mid \textbf{do}\ \text{block}$$



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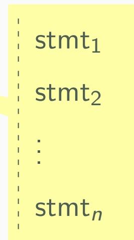


Q: Is this grammar ambiguous?

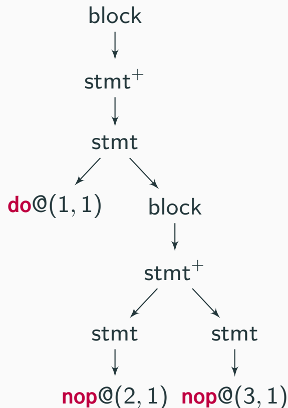
It Is Ambiguous

do

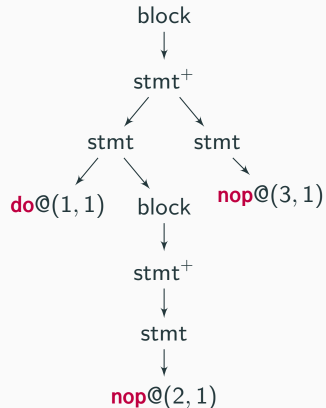
nop

nop

[**do**@(1, 1),
nop@(2, 1),
nop@(3, 1)]



parse tree t_1



parse tree t_2

Lessons

Ambiguity should be detected automatically, not manually.

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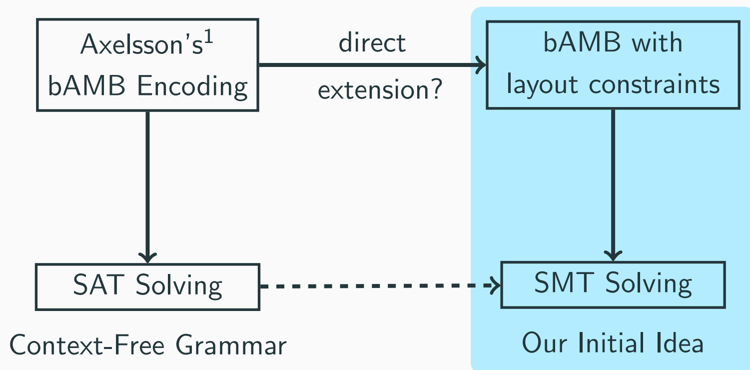
- Q: But isn't ambiguity undecidable?
- A: Find an incomplete but useful way.

Bounded Ambiguity Checking

Definition (Bounded ambiguity)
Given a layout-sensitive grammar G, is there a sentence of length up to k that has multiple parse trees?

Bounded Ambiguity Checking



[1] Axelsson, Roland, Keijo Heljanko, and Martin Lange. "Analyzing context-free grammars using an incremental SAT solver." ICALP 2008, Reykjavik, Iceland, July 7-11, 2008, Proceedings, Part II 35. Springer Berlin Heidelberg, 2008.

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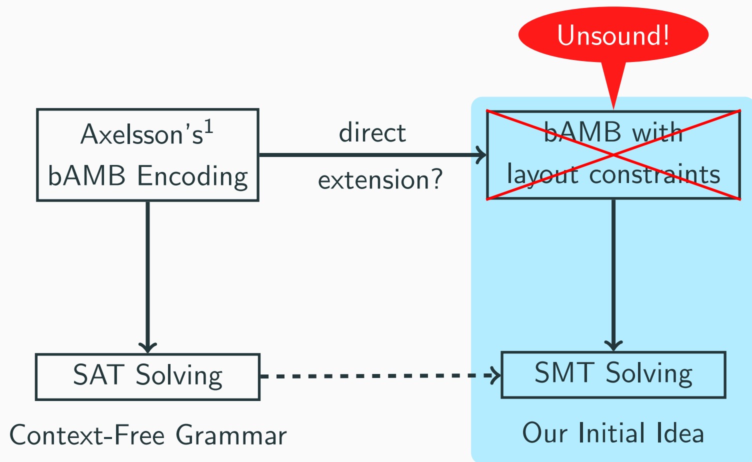

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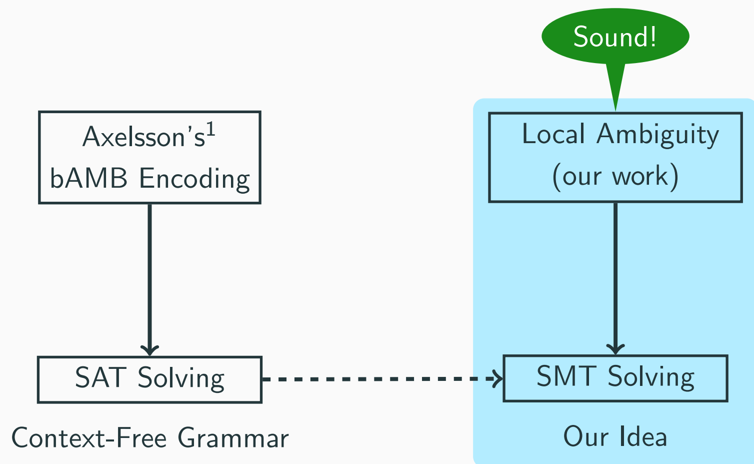

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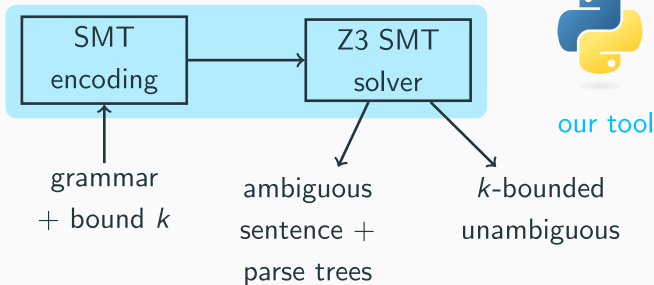
Framework for Layout-Sensitive Ambiguity Detection



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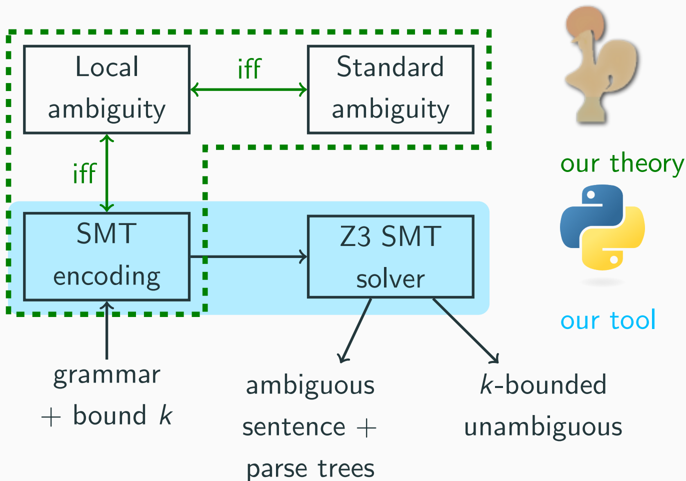
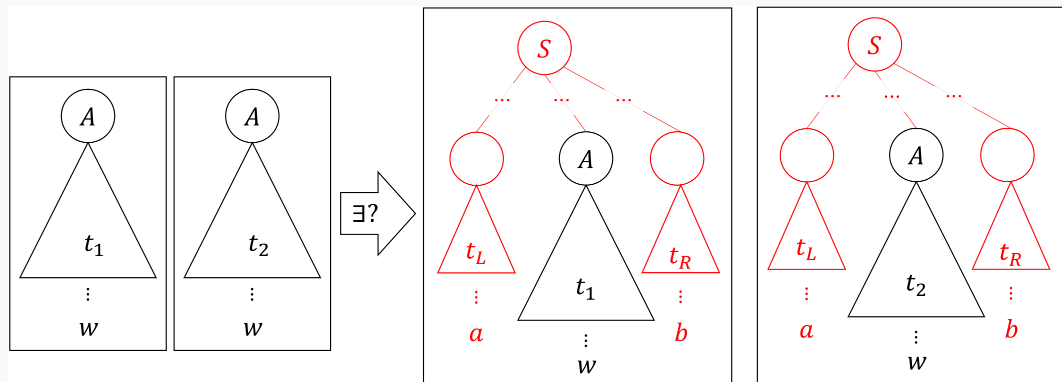


our theory

our tool

Axelsson's bAMB Encoding



bAMB = subtrees differ on the first level

Axelsson's bAMB is Unsound

The following grammer is unambiguous:

$$S \to (\!| A |\!) \qquad A \to B_1 \qquad A \to B_2$$

$$B_1 \to C \parallel C \qquad B_2 \to C\ C \qquad C \to c$$

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$(\!|\cdot|\!)$: all tokens derived are on the same line

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$\cdot \parallel \cdot$: first tokens produced by both sides lie at the same column

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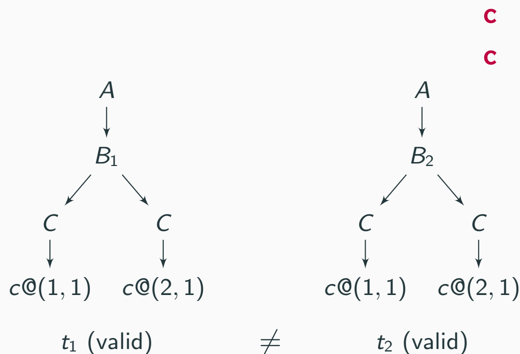
... but the following example satisfies bAMB:

c

c



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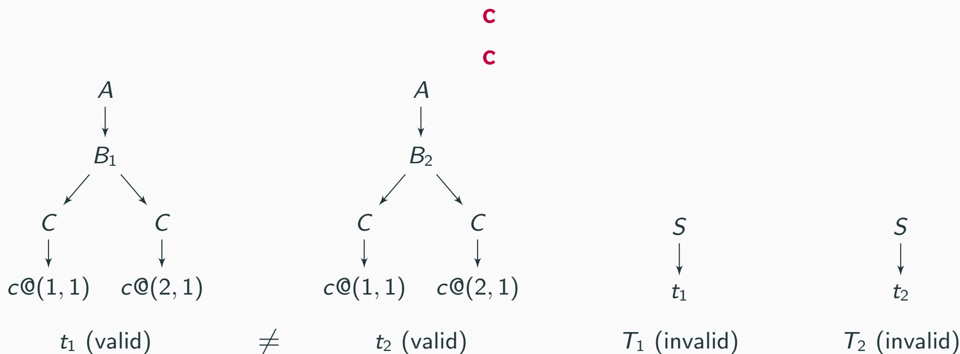

Why This Happens to Layout-Sensitive Grammars?

The complete parse trees can still be malformed/invalid,
even if valid ambiguous subtrees exist!

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Solution: drop subtrees that are unreachable from the start symbol.

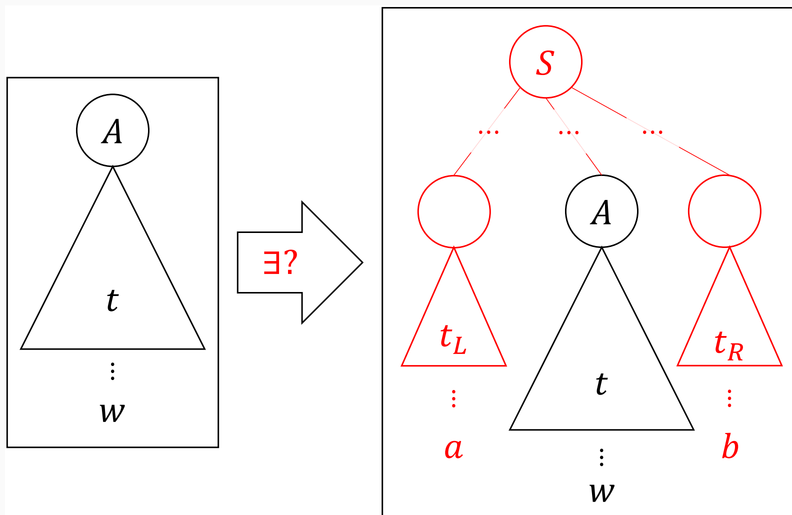
Reachability (for Context-Free Grammars)

Definition
Nonterminal A is reachable[2] in context-free grammar G from symbol S if there exists a derivation $S \Rightarrow_* \alpha A \beta$ for some sentential forms α and β.

[2] Hopcroft, John E., Rajeev Motwani, and Jeffrey D. Ullman. 2006. "Introduction to automata theory, languages, and computation (3rd Edition)." Addison-Wesley Longman Publishing Co., Inc., USA.

Reachability (for Layout-Sensitive Grammars)

Given a subtree with A as root and derives sentence w



Local Ambiguity

Intuitively, local ambiguity is essentially

$$\text{bAMB} \land \text{reachability}$$

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$$\text{bAMB} \wedge \text{reachability}$$

Theorem
(Equivalence between (standard) ambiguity and local ambiguity)
(A, w) *is locally ambiguous iff the derivation* $A \Rightarrow_* w$ *is ambiguous.*

Encoding Local Ambiguity

$$\Phi(k) \triangleq \Phi_D(k) \wedge \Phi_R^{\varepsilon}(k) \wedge \Phi_R^{\not\varepsilon}(k)$$
$$\wedge \bigvee_{H \in N} \left((\mathcal{R}_{\varepsilon}^{H} \wedge \Phi_{\text{multi}}(H,0,0)) \vee \bigvee_{\substack{0<\delta \\ x+\delta \leq k}} (\mathcal{R}_{x,\delta}^{H} \wedge \Phi_{\text{multi}}(H,x,\delta)) \right)$$

- $\Phi_D(k)$: the sentence can be derived in given grammar
- $\Phi_{\text{multi}}(H,x,\delta)$: at least two parse trees exist

} bAMB

- $\Phi_R^{\varepsilon}(k)$, $\Phi_R^{\not\varepsilon}(k)$, $\mathcal{R}_{\varepsilon}^{H}$, $\mathcal{R}_{x,\delta}^{H}$: reachability conditions

Soundness & Completeness of Encoding

Our encoding $\Phi(k)$ is bounded sound and complete:

Soundness Any sentence w that satisfies $\Phi(k) \Rightarrow w$ is ambiguous.

Completeness If there exists an ambiguous sentence of length k, then $\Phi(k)$ is satisfiable.

Take-Home Messages

- Ambiguity is important in language design.
- Proof assistant can help with finding design flaws in encoding.
- SMT solving is powerful for formal language research.

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Check our website:

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